Hence

$$R_H = \frac{V_H w}{BI} \tag{5-60}$$

If conduction is due primarily to charges of one sign, the conductivity σ is related to the mobility μ by Eq. (3-3), or

$$\sigma = \rho\mu \tag{5-61}$$

If the conductivity is measured together with the Hall coefficient, the mobility can be determined from

$$\mu = \sigma R_H \tag{5-62}$$

We have assumed in the foregoing discussion that all particles travel with the mean drift speed v. Actually, the current carriers have a random thermal distribution in speed. If this distribution is taken into account, it is found that Eq. (5-60) remains valid provided that R_H is defined by $3\pi/8\rho$. Also, Eq. (5-62) must be modified to $\mu = (8\sigma/3\pi)R_H$.

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6/ SEMICONDUCTOR-DIODE CHARACTERISTICS

In this chapter we demonstrate that if a junction is formed between a sample of p-type and one of n-type semiconductor, this combination possesses the properties of a rectifier. The volt-ampere characteristics of such a junction are derived. Electron and hole currents as a function of distance are studied in detail. The capacitance across the junction is calculated.

Although the transistor is a triode semiconductor, it may be considered as one diode biased by the current from a second diode. Hence most of the theory developed in this chapter is utilized later in connection with our study of the transistor.

6-1 QUALITATIVE THEORY OF THE p-n JUNCTION[1]

If donor impurities are introduced into one side and acceptors into the other side of a single crystal of a semiconductor, say, germanium, a p-n junction is formed. Such a system is illustrated in Fig. 6-1a. The donor ion is indicated schematically by a plus sign because, after this impurity atom "donates" an electron, it becomes a positive ion. The acceptor ion is indicated by a minus sign because, after this atom "accepts" an electron, it becomes a negative ion. Initially, there are nominally only p-type carriers to the left of the junction and only n-type carriers to the right. Because there is a density gradient across the junction, holes will diffuse to the right across the junction, and electrons to the left.

As a result of the displacement of these charges, an electric field will appear across the junction. Equilibrium will be established when the field becomes large enough to restrain the process of diffusion. The general shape of the charge distribution may be as illustrated in

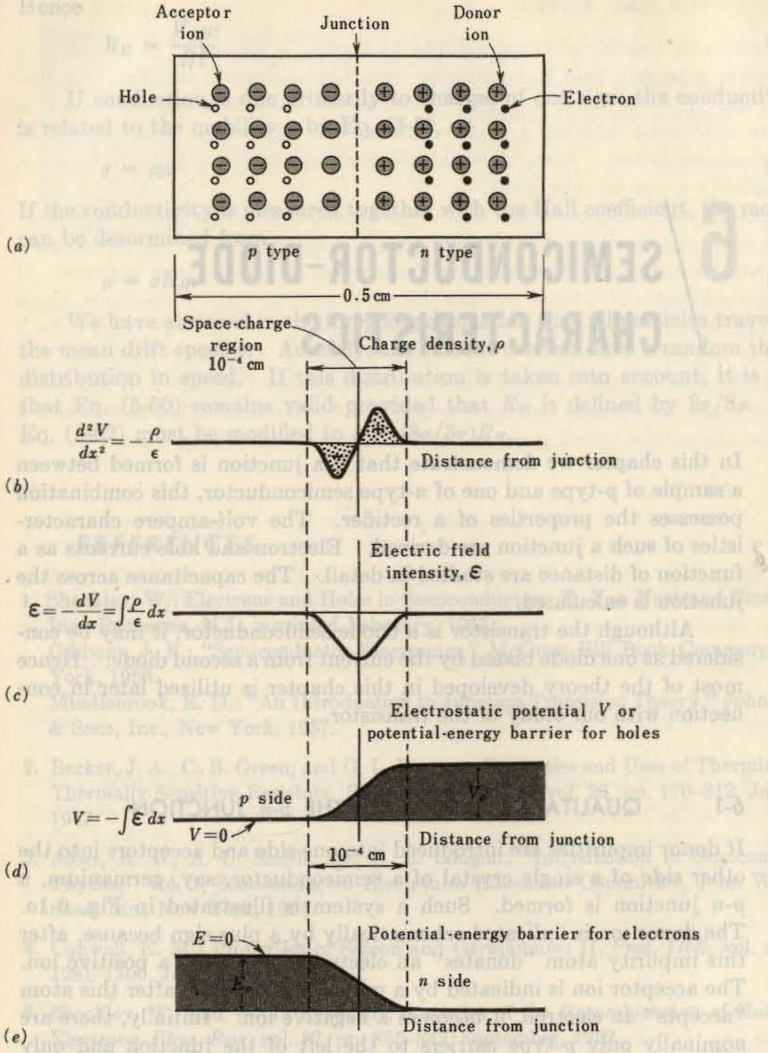

Fig. 6-1 A schematic diagram of a *p-n* junction, including the charge density, electric field intensity, and potential-energy barriers at the junction. (Not drawn to scale.)

Fig. 6-1*b*. The electric charges are confined to the neighborhood of the junction, and consist of immobile ions. We see that the positive holes which neutralized the acceptor ions near the junction in the *p*-type germanium have disappeared as a result of combination with electrons which have diffused across the junction. Similarly, the neutralizing electrons in the *n*-type germanium have combined with holes which have crossed the junction from the *p* material. The unneutralized ions in the neighborhood of the junction are referred to as *uncovered charges.* Since the region of the junction is depleted of mobile charges, it is called the *depletion region,* the *space-charge region,* or the *transition region.* The thickness of this region is of the order of

$$10^{-4}\text{ cm} = 10^{-6}\text{ m} = 1\text{ micron}$$

The electric field intensity in the neighborhood of the junction is indicated in Fig. 6-1*c*. Note that this curve is the integral of the density function ρ in Fig. 6-1*b*. The electrostatic-potential variation in the depletion region is shown in Fig. 6-1*d*, and is the negative integral of the function $\mathcal{E}$ of Fig. 6-1*c*. This variation constitutes a potential-energy barrier against the further diffusion of holes across the barrier. The form of the potential-energy barrier against the flow of electrons from the *n* side across the junction is shown in Fig. 6-1*e*. It is similar to that shown in Fig. 6-1*d*, except that it is inverted, since the charge on an electron is negative.

The necessity for the existence of a potential barrier called the *contact,* or *diffusion, potential* is now considered further. Under open-circuited conditions the net hole current must be zero. If this statement were not true, the hole density at one end of the semiconductor would continue to increase indefinitely with time, a situation which is obviously physically impossible. Since the concentration of holes in the *p* side is much greater than that in the *n* side, a very large diffusion current tends to flow across the junction from the *p* to the *n* material. Hence an electric field must build up across the junction in such a direction that a drift current will tend to flow across the junction from the *n* to the *p* side in order to counterbalance the diffusion current. This equilibrium condition of zero resultant hole current allows us to calculate the height of the potential barrier V_o [Eq. (6-8)] in terms of the donor and acceptor concentrations. The numerical value for V_o is of the order of magnitude of a few tenths of a volt.

6-2 THE *p-n* JUNCTION AS A DIODE

The essential electrical characteristic of a *p-n* junction is that it constitutes a diode which permits the easy flow of current in one direction but restrains the flow in the opposite direction. We consider now, qualitatively, how this diode action comes about.

Reverse Bias In Fig. 6-2, a battery is shown connected across the terminals of a *p-n* junction. The negative terminal of the battery is con-

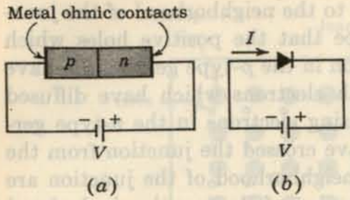

Fig. 6-2 (a) A p-n junction biased in the reverse direction. (b) The rectifier symbol is used for the p-n diode.

nected to the p side of the junction, and the positive terminal to the n side. The polarity of connection is such as to cause both the holes in the p type and the electrons in the n type to move away from the junction. Consequently, the region of negative-charge density is spread to the left of the junction (Fig. 6-1b), and the positive-charge-density region is spread to the right. However, this process cannot continue indefinitely, because in order to have a steady flow of holes to the left, these holes must be supplied across the junction from the n-type germanium. And there are very few holes in the n-type side. Hence, nominally, zero current results. Actually, a small current does flow because a small number of hole-electron pairs are generated throughout the crystal as a result of thermal energy. The holes so formed in the n-type germanium will wander over to the junction. A similar remark applies to the electrons thermally generated in the p-type germanium. This small current is the diode *reverse saturation current*, and its magnitude is designated by I_o. This reverse current will increase with increasing temperature [Eq. (6-28)], and hence the back resistance of a crystal diode decreases with increasing temperature.

The mechanism of conduction in the reverse direction may be described alternatively in the following way: When no voltage is applied to the p-n diode, the potential barrier across the junction is as shown in Fig. 6-1d. When a voltage V is applied to the diode in the direction shown in Fig. 6-2, the height of the potential-energy barrier is increased by the amount eV. This increase in the barrier height serves to reduce the flow of majority carriers (i.e., holes in p type and electrons in n type). However, the minority carriers (i.e., electrons in p type and holes in n type), since they fall down the potential-energy hill, are uninfluenced by the increased height of the barrier. The applied voltage in the direction indicated in Fig. 6-2 is called the *reverse*, or *blocking*, *bias*.

Forward Bias An external voltage applied with the polarity shown in Fig. 6-3 (opposite to that indicated in Fig. 6-2) is called a *forward* bias. An ideal p-n diode has zero ohmic voltage drop across the body of the crystal. For such a diode the height of the potential barrier at the junction will be lowered by the applied forward voltage V. The equilibrium initially established between the forces tending to produce diffusion of majority carriers and the restraining influence of the potential-energy barrier at the junction will be disturbed. Hence, for a forward bias, the holes cross the junction from the p type to the n type, and the electrons cross the junction in the opposite direction. These majority carriers can then travel around the closed circuit, and a relatively large current will flow.

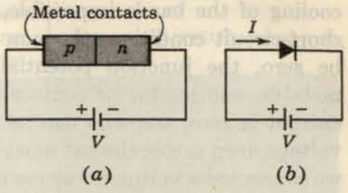

Fig. 6-3 (a) A p-n junction biased in the forward direction. (b) The rectifier symbol is used for the p-n diode.

Ohmic Contacts[1] In Fig. 6-2 (6-3) we show an external reverse (forward) bias applied to a p-n diode. We have assumed that the external bias voltage appears directly across the junction and has the effect of raising (lowering) the electrostatic potential across the junction. In order to justify this assumption we must specify how electric contact is made to the semiconductor from the external bias circuit. In Figs. 6-2 and 6-3 we indicate metal contacts with which the homogeneous p-type and n-type materials are provided. We thus see that we have introduced two metal-semiconductor junctions, one at each end of the diode. We naturally expect a contact potential to develop across these additional junctions. However, we shall assume that the metal-semiconductor contacts shown in Figs. 6-2 and 6-3 have been manufactured in such a way that they are nonrectifying. In other words, the contact potential across these junctions is approximately independent of the direction and magnitude of the current. A contact of this type is referred to as an *ohmic contact*.

We are now in a position to justify our assumption that the entire applied voltage appears as a *change* in the height of the potential barrier. Inasmuch as the metal-semiconductor contacts are low-resistance ohmic contacts and the voltage drop across the bulk of the crystal is neglected, approximately the entire applied voltage will indeed appear as a change in the height of the potential barrier at the p-n junction.

The Short-circuited and Open-circuited p-n Junction If the voltage V in Fig. 6-2 or 6-3 were set equal to zero, the p-n junction would be short-circuited. Under these conditions, as we show below, no current can flow ($I = 0$) and the electrostatic potential V_o remains unchanged and equal to the value under open-circuit conditions. If there were a current ($I \neq 0$), the metal would become heated. Since there is no external source of energy available, the energy required to heat the metal wire would have to be supplied by the p-n bar. The semiconductor bar, therefore, would have to cool off. Clearly, under thermal equilibrium the simultaneous heating of the metal and cooling of the bar is impossible, and we conclude that $I = 0$. Since under short-circuit conditions the sum of the voltages around the closed loop must be zero, the junction potential V_o must be exactly compensated by the metal-to-semiconductor contact potentials at the ohmic contacts. Since the current is zero, the wire can be cut without changing the situation, and the voltage drop across the cut must remain zero. If in an attempt to measure V_o we connected a voltmeter across the cut, the voltmeter would read zero voltage. In other words, it is not possible to measure contact difference of potential directly with a voltmeter.

Large Forward Voltages Suppose that the forward voltage V in Fig. 6-3 is increased until V approaches V_o. If V were equal to V_o, the barrier would disappear and the current could be arbitrarily large, exceeding the rating of the diode. As a practical matter we can never reduce the barrier to zero because, as the current increases without limit, the bulk resistance of the crystal, as well as the resistance of the ohmic contacts, will limit the current. Therefore it is no longer possible to assume that all the voltage V appears as a change across the p-n junction. We conclude that, as the forward voltage V becomes comparable with V_o, the current through a real p-n diode will be governed by the ohmic-contact resistances and the crystal bulk resistance. Thus the volt-ampere characteristic becomes approximately a straight line.

6-3 BAND STRUCTURE OF AN OPEN-CIRCUITED *p-n* JUNCTION

As in the previous section, we here consider that a p-n junction is formed by placing p- and n-type materials in intimate contact on an atomic scale. Under these conditions the Fermi level must be constant throughout the specimen at equilibrium. If this were not so, electrons on one side of the junction would have an average energy higher than those on the other side, and there would be a transfer of electrons and energy until the Fermi levels in the two sides did line up. In Sec. 5-6 it is verified that the Fermi level E_F is closer to the conduction band edge E_{Cn} in the n-type material and closer to the valence band edge E_{Vp} in the p side. Clearly, then, the conduction band edge E_{Cp} in the p material cannot be at the same level as E_{Cn}, nor can the valence band edge E_{Vn} in the n side line up with E_{Vp}. Hence the energy-band diagram for a p-n junction appears as shown in Fig. 6-4, where a shift in energy levels E_o is indicated. Note that

$$E_o = E_{Cp} - E_{Cn} = E_{Vp} - E_{Vn} = E_1 + E_2 \tag{6-1}$$

This energy E_o represents the potential energy of the electrons at the junction, as is indicated in Fig. 6-1*e*.

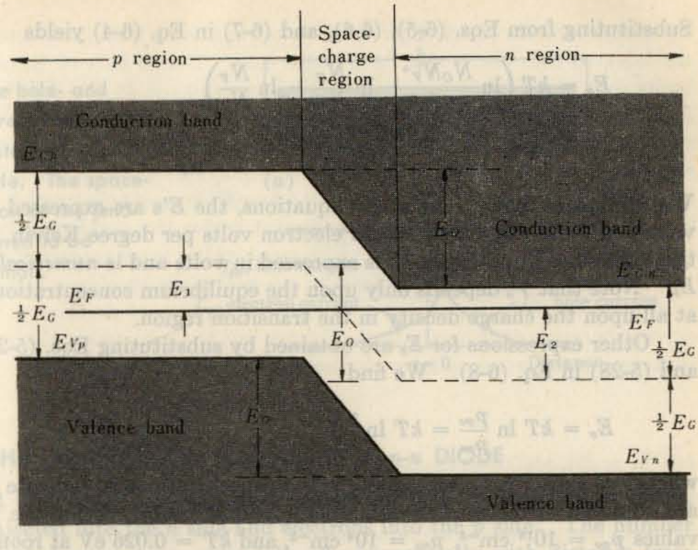

Fig. 6-4 Band diagram for a *p-n* junction under open-circuit conditions. This sketch corresponds to Fig. 6-1*e* and represents potential energy for electrons. The width of the forbidden gap is E_G in electron volts.

The Contact Difference of Potential We now obtain an expression for E_o. From Fig. 6-4 we see that

$$E_F - E_{Vp} = \tfrac{1}{2}E_G - E_1 \tag{6-2}$$

and

$$E_{Cn} - E_F = \tfrac{1}{2}E_G - E_2 \tag{6-3}$$

Adding these two equations, we obtain

$$E_o = E_1 + E_2 = E_G - (E_{Cn} - E_F) - (E_F - E_{Vp}) \tag{6-4}$$

From Eqs. (5-18) and (5-19),

$$E_G = kT \ln \frac{N_C N_V}{n_i^2} \tag{6-5}$$

From Eq. (5-30),

$$E_{Cn} - E_F = kT \ln \frac{N_C}{N_D} \tag{6-6}$$

From Eq. (5-31),

$$E_F - E_{Vp} = kT \ln \frac{N_V}{N_A} \tag{6-7}$$

Substituting from Eqs. (6-5), (6-6), and (6-7) in Eq. (6-4) yields

$$E_o = kT\left(\ln\frac{N_C N_V}{n_i^2} - \ln\frac{N_C}{N_D} - \ln\frac{N_V}{N_A}\right)$$

$$= kT\ln\left(\frac{N_C N_V}{n_i^2}\frac{N_D}{N_C}\frac{N_A}{N_V}\right) = kT\ln\frac{N_D N_A}{n_i^2} \tag{6-8}$$

We emphasize that, in the above equations, the E's are expressed in electron volts and k has the dimensions of electron volts per degree Kelvin. The contact difference in potential V_o is expressed in volts and is *numerically* equal to E_o. Note that V_o depends only upon the equilibrium concentrations, and not at all upon the charge density in the transition region.

Other expressions for E_o are obtained by substituting Eqs. (5-26), (5-27), and (5-28) in Eq. (6-8). We find

$$E_o = kT\ln\frac{p_{po}}{p_{no}} = kT\ln\frac{n_{no}}{n_{po}} \tag{6-9}$$

where the subscripts o are added to the concentrations to indicate that these are obtained under conditions of thermal equilibrium. Using the reasonable values $p_{po} = 10^{16}$ cm^{-3}, $p_{no} = 10^4$ cm^{-3}, and $kT = 0.026$ eV at room temperature, we obtain $E_o \approx 0.5$ eV.

An Alternative Derivation[2] for V_o In Sec. 6-1 we indicate that an application of the equilibrium condition of zero resultant hole current allows a calculation of V_o to be made. We now carry out such an analysis. Since the net hole current density is zero, the negative of the hole diffusion current [Eq. (5-32)] must equal the hole drift current [Eq. (3-2)], or

$$eD_p\frac{dp}{dx} = e\mu_p p\mathcal{E} \tag{6-10}$$

The Einstein relation [Eq. (5-33)] is

$$\frac{D_p}{\mu_p} = V_T \tag{6-11}$$

where the volt equivalent of temperature V_T is defined by Eq. (3-34). Substituting Eq. (6-11) in Eq. (6-10) and remembering the relationship (1-15) between field intensity and potential, we obtain

$$\frac{dp}{p} = \frac{\mathcal{E}\,dx}{V_T} = -\frac{dV}{V_T} \tag{6-12}$$

If this equation is integrated between limits which extend across the junction (Fig. 6-1d) from the p material, where the equilibrium hole concentration is p_{po}, to the n side, where the hole density is p_{no}, the result is

$$p_{po} = p_{no}\epsilon^{V_o/V_T} \tag{6-13}$$

Since $V_o/V_T = E_o/kT$, Eq. (6-13) is equivalent to Eq. (6-9).

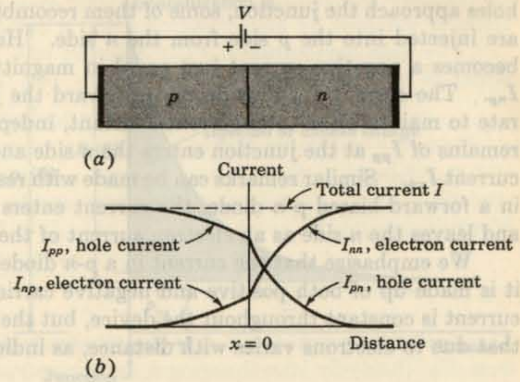

Fig. 6-5 The hole- and electron-current components vs. distance in a p-n junction diode. The space-charge region at the junction is assumed to be negligibly small.

6-4 THE CURRENT COMPONENTS IN A p-n DIODE

In Sec. 6-2 it is indicated that when a forward bias is applied to a diode, holes are injected into the n side and electrons into the p side. The number of these injected minority carriers falls off exponentially with distance from the junction [Eq. (5-50)]. Since the diffusion current of minority carriers is proportional to the concentration gradient [Eq. (5-32)], this current must also vary exponentially with distance. There are two minority currents, I_{pn} and I_{np}, and these are indicated in Fig. 6-5. The symbol† $I_{pn}(x)$ represents the hole current in the n material, and $I_{np}(x)$ indicates the electron current in the p side as a function of x.

Electrons crossing the junction at $x = 0$ from right to left constitute a current in the same direction as holes crossing the junction from left to right. Hence the total current I at $x = 0$ is

$$I = I_{pn}(0) + I_{np}(0) \tag{6-14}$$

Since the current is the same throughout a series circuit, I is independent of x, and is indicated as a horizontal line in Fig. 6-5. Consequently, in the p side, there must be a second component of current I_{pp} which, when added to I_{np}, gives the total current I. Hence this hole current in the p side I_{pp} (a majority carrier current) is given by

$$I_{pp}(x) = I - I_{np}(x) \tag{6-15}$$

This current is plotted as a function of distance in Fig. 6-5, as is also the corresponding electron current I_{nn} in the n material. This figure is drawn for an unsymmetrically doped diode, so that $I_{pn} \neq I_{np}$.

Note that deep into the p side the current is a drift (conduction) current I_{pp} of holes sustained by the small electric field in the semiconductor. As the

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† If the letters p and n both appear in a symbol, the first letter refers to the type of carrier, and the second to the type of material.

holes approach the junction, some of them recombine with the electrons, which are injected into the p side from the n side. Hence part of the current I_{pp} becomes a negative current just equal in magnitude to the diffusion current I_{np}. The current I_{pp} thus decreases toward the junction (at just the proper rate to maintain the total current constant, independent of distance). What remains of I_{pp} at the junction enters the n side and becomes the hole diffusion current I_{pn}. Similar remarks can be made with respect to current I_{nn}. Hence, in a forward-biased p-n diode, the current enters the p side as a hole current and leaves the n side as an electron current of the same magnitude.

We emphasize that the current in a p-n diode is bipolar in character since it is made up of both positive and negative carriers of electricity. The total current is constant throughout the device, but the proportion due to holes and that due to electrons varies with distance, as indicated in Fig. 6-5.

6-5 QUANTITATIVE THEORY OF THE p-n DIODE CURRENTS

We now derive the expression for the total current as a function of the applied voltage (the volt-ampere characteristic). In the discussion to follow we neglect the depletion-layer thickness, and hence assume that the barrier width is zero. If a forward bias is applied to the diode, holes are injected from the p side into the n material. The concentration p_n of holes in the n side is increased above its thermal-equilibrium value p_{no} and, as indicated in Eq. (5-52), is given by

$$p_n(x) = p_{no} + P_n(0)\epsilon^{-x/L_p} \tag{6-16}$$

where the parameter L_p is called the *diffusion length for holes* in the n material, and the *injected*, or *excess*, concentration at $x = 0$ is

$$P_n(0) = p_n(0) - p_{no} \tag{6-17}$$

These several hole-concentration components are indicated in Fig. 6-6, which shows the exponential decrease of the density $p_n(x)$ with distance x into the n material.

From Eq. (5-32) the diffusion hole current in the n side is given by

$$I_{pn} = -AeD_p\frac{dp_n}{dx} \tag{6-18}$$

Taking the derivative of Eq. (6-16) and substituting in Eq. (6-18), we obtain

$$I_{pn}(x) = \frac{AeD_pP_n(0)}{L_p}\epsilon^{-x/L_p} \tag{6-19}$$

This equation verifies that the hole current decreases exponentially with distance. The dependence of I_{pn} upon applied voltage is contained implicitly in the factor $P_n(0)$ because the injected concentration is a function of voltage. We now find the dependence of $P_n(0)$ upon V.

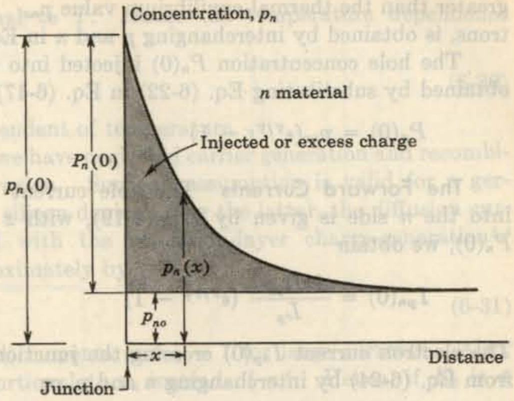


Fig. 6-6 Defining the several components of hole concentration in the n side of a forward-biased diode. The diagram is not drawn to scale since $p_n(0) \gg p_{no}$.

The Law of the Junction If the hole concentrations at the edges of the space-charge region are p_p and p_n in the p and n materials, respectively, and if the barrier potential across this depletion layer is V_B, then

$$p_p = p_n\epsilon^{V_B/V_T} \tag{6-20}$$

This is the Boltzmann relationship of kinetic gas theory. It is valid[2] even under nonequilibrium conditions as long as the net hole current is small compared with the diffusion or the drift hole current. Under this condition, called *low-level injection*, we may to a good approximation again equate the magnitudes of the diffusion and drift currents. Starting with Eqs. (6-10) and (6-12) and integrating over the depletion layer, Eq. (6-20) is obtained.

If we apply Eq. (6-20) to the case of an open-circuited p-n junction, then $p_p = p_{po}$, $p_n = p_{no}$, and $V_B = V_o$. Substituting these values in Eq. (6-20), it reduces to Eq. (6-13), from which we obtain the contact potential V_o.

Consider now a junction biased in the forward direction by an applied voltage V. Then the barrier voltage V_B is decreased from its equilibrium value V_o by the amount V, or $V_B = V_o - V$. The hole concentration throughout the p region is constant and equal to the thermal equilibrium value, or $p_p = p_{po}$. The hole concentration varies with distance into the n side, as indicated in Fig. 6-6. At the edge of the depletion layer, $x = 0$, $p_n = p_n(0)$. The Boltzmann relation (6-20) is, for this case,

$$p_{po} = p_n(0)\epsilon^{(V_o-V)/V_T} \tag{6-21}$$

Combining this equation with Eq. (6-13), we obtain

$$p_n(0) = p_{no}\epsilon^{V/V_T} \tag{6-22}$$

This boundary condition is called the *law of the junction*. It indicates that, for a forward bias ($V > 0$), the hole concentration $p_n(0)$ at the junction is greater than the thermal-equilibrium value p_{no}. A similar law, valid for electrons, is obtained by interchanging p and n in Eq. (6-22).

The hole concentration $P_n(0)$ injected into the n side at the junction is obtained by substituting Eq. (6-22) in Eq. (6-17), yielding

$$P_n(0) = p_{no}(\epsilon^{V/V_T} - 1) \qquad (6\text{-}23)$$

The Forward Currents The hole current $I_{pn}(0)$ crossing the junction into the n side is given by Eq. (6-19), with $x = 0$. Using Eq. (6-23) for $P_n(0)$, we obtain

$$I_{pn}(0) = \frac{AeD_p p_{no}}{L_p}(\epsilon^{V/V_T} - 1) \qquad (6\text{-}24)$$

The electron current $I_{np}(0)$ crossing the junction into the p side is obtained from Eq. (6-24) by interchanging n and p, or

$$I_{np}(0) = \frac{AeD_n n_{po}}{L_n}(\epsilon^{V/V_T} - 1) \qquad (6\text{-}25)$$

Finally, from Eq. (6-14), the total diode current I is the sum of $I_{pn}(0)$ and $I_{np}(0)$, or

$$I = I_o(\epsilon^{V/V_T} - 1) \qquad (6\text{-}26)$$

where

$$I_o \equiv \frac{AeD_p p_{no}}{L_p} + \frac{AeD_n n_{po}}{L_n} \qquad (6\text{-}27)$$

If W_p and W_n are the widths of the p and n materials, respectively, the above derivation has implicitly assumed that $W_p \gg L_p$ and $W_n \gg L_n$. If, as sometimes happens in a practical diode, the widths are much smaller than the diffusion lengths, the expression for I_o remains valid provided that L_p and L_n are replaced by W_p and W_n, respectively (Prob. 6-9).

The Reverse Saturation Current In the foregoing discussion a positive value of V indicates a forward bias. The derivation of Eq. (6-26) is equally valid if V is negative, signifying an applied reverse-bias voltage. For a reverse bias whose magnitude is large compared with V_T (~26 mV at room temperature), $I \to -I_o$. Hence I_o is called the *reverse saturation current.* Combining Eqs. (5-27), (5-28), and (6-27), we obtain

$$I_o = Ae\left(\frac{D_p}{L_pN_D} + \frac{D_n}{L_nN_A}\right)n_i^2 \qquad (6\text{-}28)$$

where n_i^2 is given by Eq. (5-23),

$$n_i^2 = A_oT^3\epsilon^{-E_{GO}/kT} = A_oT^3\epsilon^{-V_{GO}/V_T} \qquad (6\text{-}29)$$

where V_{GO} is a voltage which is numerically equal to the forbidden-gap energy E_{GO} in electron volts, and V_T is the volt equivalent of temperature [Eq. (3-34)]. For germanium the diffusion constants D_p and D_n vary approximately[3] inversely proportional to T. Hence the temperature dependence of I_o is

$$I_o = K_1T^2\epsilon^{-V_{GO}/V_T} \qquad (6\text{-}30)$$

where K_1 is a constant independent of temperature.

Throughout this section we have neglected carrier generation and recombination in the space-charge region. Such an assumption is valid for a germanium diode, but not for a silicon device. For the latter, the diffusion current is negligible compared with the transition-layer charge-generation[3,4] current, which is given approximately by

$$I = I_o(\epsilon^{V/\eta V_T} - 1) \qquad (6\text{-}31)$$

where $\eta \approx 2$ for small (rated) currents and $\eta \approx 1$ for large currents. Also, I_o is now found to be proportional to n_i instead of n_i^2. Hence, if K_2 is a constant,

$$I_o = K_2T^{1.5}\epsilon^{-V_{GO}/2V_T} \qquad (6\text{-}32)$$

The practical implications of these diode equations are given in the following sections.

6-6 THE VOLT-AMPERE CHARACTERISTIC

The discussion of the preceding section indicates that, for a p-n junction, the current I is related to the voltage V by the equation

$$I = I_o(\epsilon^{V/\eta V_T} - 1) \qquad (6\text{-}33)$$

A positive value of I means that current flows from the p to the n side. The diode is forward-biased if V is positive, indicating that the p side of the junction is positive with respect to the n side. The symbol η is unity for germanium and is approximately 2 for silicon.

The symbol V_T stands for the volt equivalent of temperature, and is given by Eq. (3-34), repeated here for convenience:

$$V_T \equiv \frac{T}{11{,}600} \qquad (6\text{-}34)$$

At room temperature ($T = 300°$K), $V_T = 0.026$ V $= 26$ mV.

The form of the volt-ampere characteristic described by Eq. (6-33) is shown in Fig. 6-7*a*. When the voltage V is positive and several times V_T, the unity in the parentheses of Eq. (6-33) may be neglected. Accordingly, except for a small range in the neighborhood of the origin, the current increases exponentially with voltage. When the diode is reverse-biased and $|V|$ is several times V_T, $I \approx -I_o$. The reverse current is therefore constant, independent of the applied reverse bias. Consequently, I_o is referred to as the *reverse saturation current.*

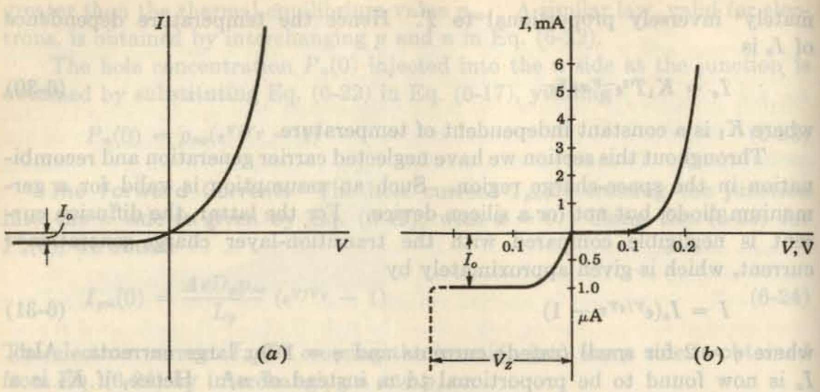


Fig. 6-7 (a) The volt-ampere characteristic of an ideal *p-n* diode. (b) The volt-ampere characteristic for a germanium diode redrawn to show the order of magnitude of currents. Note the expanded scale for reverse currents. The dashed portion indicates breakdown at V_Z.

For the sake of clarity, the current I_o in Fig. 6-7*a* has been greatly exaggerated in magnitude. Ordinarily, the range of forward currents over which a diode is operated is many orders of magnitude larger than the reverse saturation current. In order to display forward and reverse characteristics conveniently, it is necessary, as in Fig. 6-7*b*, to use two different current scales. The volt-ampere characteristic shown in that figure has a forward current scale in milliamperes and a reverse scale in microamperes.

The dashed portion of the curve of Fig. 6-7*b* indicates that, at a reverse-biasing voltage V_Z, the diode characteristic exhibits an abrupt and marked departure from Eq. (6-33). At this critical voltage a large reverse current flows, and the diode is said to be in the *breakdown* region, discussed in Sec. 6-12.

The Cutin Voltage V_γ Both silicon and germanium diodes are commercially available. A number of differences between these two types are relevant in circuit design. The difference in volt-ampere characteristics is brought out in Fig. 6-8. Here are plotted the forward characteristics at room temperature of a general-purpose germanium switching diode and a general-purpose silicon diode, the 1N270 and 1N3605, respectively. The diodes have comparable current ratings. A noteworthy feature in Fig. 6-8 is that there exists a *cutin*, *offset*, *break-point*, or *threshold* voltage V_γ below which the current is very small (say, less than 1 percent of maximum rated value). Beyond V_γ the current rises very rapidly. From Fig. 6-8 we see that V_γ is approximately 0.2 V for germanium and 0.6 V for silicon.

Note that the break in the silicon-diode characteristic is offset about 0.4 V with respect to the break in the germanium-diode characteristic. The reason for this difference is to be found, in part, in the fact that the reverse saturation current in a germanium diode is normally larger by a factor of about 1,000 than the reverse saturation current in a silicon diode of comparable ratings. Thus, if I_o is in the range of microamperes for a germanium diode, I_o will be in the range of nanoamperes for a silicon diode.

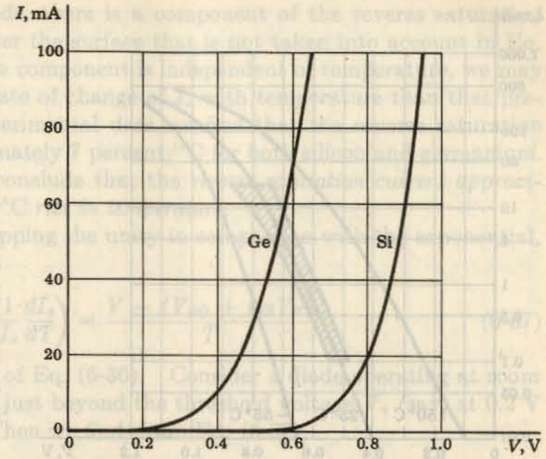


Fig. 6-8 The forward volt-ampere characteristics of a germanium (1N270) and a silicon (1N3605) diode at 25°C.

Since $\eta = 2$ for small currents in silicon, the current increases as $\epsilon^{V/2V_T}$ for the first several tenths of a volt and increases as ϵ^{V/V_T} only at higher voltages. This initial smaller dependence of the current on voltage accounts for the further delay in the rise of the silicon characteristic.

Logarithmic Characteristic It is instructive to examine the family of curves for the silicon diodes shown in Fig. 6-9. A family for a germanium diode of comparable current rating is quite similar, with the exception that corresponding currents are attained at lower voltage.

From Eq. (6-33), assuming that V is several times V_T, so that we may drop the unity, we have $\log I = \log I_o + 0.434V/\eta V_T$. We therefore expect in Fig. 6-9, where $\log I$ is plotted against V, that the plots will be straight lines. We do indeed find that at low currents the plots are linear and correspond to $\eta \approx 2$. At large currents an increment of voltage does not yield as large an increase of current as at low currents. The reason for this behavior is to be found in the ohmic resistance of the diode. At low currents the ohmic drop is negligible and the externally impressed voltage simply decreases the potential barrier at the *p-n* junction. At high currents the externally

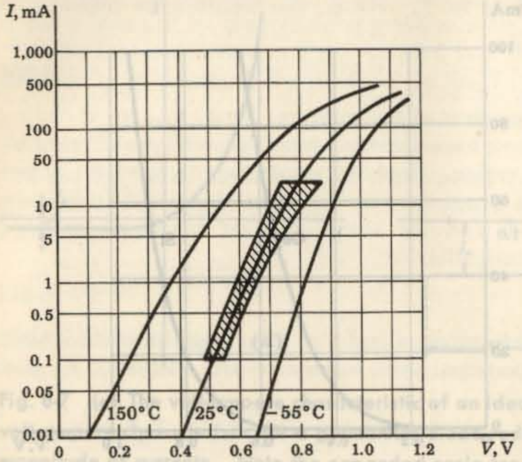


Fig. 6-9 Volt-ampere characteristics at three different temperatures for a silicon diode (planar epitaxial passivated types 1N3605, 1N3606, 1N3608, and 1N3609). The shaded area indicates 25°C limits of controlled conductance. Note that the vertical scale is logarithmic and encompasses a current range of 50,000. (Courtesy of General Electric Company.)

impressed voltage is called upon principally to establish an electric field to overcome the ohmic resistance of the semiconductor material. Therefore, at high currents, the diode behaves more like a resistor than a diode, and the current increases linearly rather than exponentially with applied voltage.

6-7 THE TEMPERATURE DEPENDENCE OF *p-n* CHARACTERISTICS

Let us inquire into the diode voltage variation with temperature at fixed current. This variation may be calculated from Eq. (6-33), where the temperature is contained implicitly in V_T and also in the reverse saturation current. The dependence of I_o on temperature T is, from Eqs. (6-30) and (6-32), given approximately by

$$I_o = KT^m \epsilon^{-V_{GO}/\eta V_T} \tag{6-35}$$

where K is a constant and eV_{GO} (e is the magnitude of the electronic charge) is the forbidden-gap energy in joules:

For Ge: $\eta = 1$ $m = 2$ $V_{GO} = 0.785$ V
For Si: $\eta = 2$ $m = 1.5$ $V_{GO} = 1.21$ V

Taking the derivative of the logarithm of Eq. (6-35), we find

$$\frac{1}{I_o}\frac{dI_o}{dT} = \frac{d(\ln I_o)}{dT} = \frac{m}{T} + \frac{V_{GO}}{\eta T V_T} \tag{6-36}$$

At room temperature, we deduce from Eq. (6-36) that $d(\ln I_o)/dT = 0.08°\text{C}^{-1}$ for Si and $0.11°\text{C}^{-1}$ for Ge. The performance of commercial diodes is only approximately consistent with these results. The reason for the discrepancy is that, in a physical diode, there is a component of the reverse saturation current due to leakage over the surface that is not taken into account in Eq. (6-35). Since this leakage component is independent of temperature, we may expect to find a smaller rate of change of I_o with temperature than that predicted above. From experimental data we find that the reverse saturation current increases approximately 7 percent/°C for both silicon and germanium. Since $(1.07)^{10} \approx 2.0$, we conclude that the *reverse saturation current approximately doubles for every* 10°C *rise in temperature.*

From Eq. (6-33), dropping the unity in comparison with the exponential, we find, for constant I,

$$\frac{dV}{dT} = \frac{V}{T} - \eta V_T\left(\frac{1}{I_o}\frac{dI_o}{dT}\right) = \frac{V - (V_{GO} + m\eta V_T)}{T} \tag{6-37}$$

where use has been made of Eq. (6-36). Consider a diode operating at room temperature (300°K) and just beyond the threshold voltage V_γ (say, at 0.2 V for Ge and 0.6 for Si). Then we find, from Eq. (6-37),

$$\frac{dV}{dT} = \begin{cases} -2.1\ \text{mV/°C} & \text{for Ge} \\ -2.3\ \text{mV/°C} & \text{for Si} \end{cases} \tag{6-38}$$

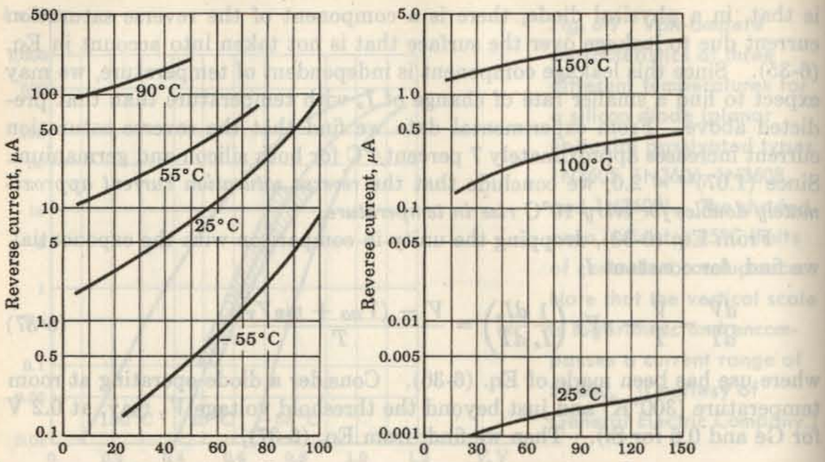
Since these data are based on "average characteristics," it might be well for conservative design to assume a value of

$$\frac{dV}{dT} = -2.5\ \text{mV/°C} \tag{6-39}$$

for either Ge or Si at room temperature. Note from Eq. (6-37) that $|dV/dT|$ decreases with increasing T.

The temperature dependence of forward voltage is given in Eq. (6-37) as the difference between two terms. The positive term V/T on the right-hand side results from the temperature dependence of V_T. The negative term results from the temperature dependence of I_o, and does not depend on the voltage V across the diode. The equation predicts that for increasing V, dV/dT should become less negative, reach zero at $V = V_{GO} + m\eta V_T$, and thereafter reverse sign and go positive. This behavior is regularly exhibited by diodes. Normally, however, the reversal takes place at a current which is higher than the maximum rated current. The curves of Fig. 6-9 also suggest this behavior. At high voltages the horizontal separation between curves of different temperatures is smaller than at low voltages.

Typical reverse characteristics of germanium and silicon diodes are given in Fig. 6-10*a* and *b*. Observe the very pronounced dependence of current on reverse voltage, a result which is not consistent with our expectation of a constant saturated reverse current. This increase in I_o results from leakage across the surface of the diode, and also from the additional fact that new current carriers may be generated by collision in the transition region at the junction. On the other hand, there are many commercially available diodes, both germanium and silicon, that do exhibit a fairly constant reverse current with



(*a*) (*b*)

Fig. 6-10 Examples of diodes which do not exhibit a constant reverse saturation current. (*a*) Germanium diode 1N270; (*b*) silicon 1N461. (Courtesy of Raytheon Company.)

increasing voltage. The much larger value of I_o for a germanium than for a silicon diode, to which we have previously referred, is apparent in comparing Fig. 6-10*a* and *b*. Since the temperature dependence is approximately the same in both types of diodes, at elevated temperatures the germanium diode will develop an excessively large reverse current, whereas for silicon, I_o will be quite modest. Thus we can see that for Ge in Fig. 6-10 an increase in temperature from room temperature (25°C) to 90°C increases the reverse current to hundreds of microamperes, although in silicon at 100°C the reverse current has increased only to some tenths of a microampere.

6-8 DIODE RESISTANCE

The static resistance R of a diode is defined as the ratio V/I of the voltage to the current. At any point on the volt-ampere characteristic of the diode (Fig. 6-7), the resistance R is equal to the reciprocal of the slope of a line joining the operating point to the origin. The static resistance varies widely with V and I and is not a useful parameter. The rectification property of a diode is indicated on the manufacturer's specification sheet by giving the maximum forward voltage V_F required to attain a given forward current I_F and also the maximum reverse current I_R at a given reverse voltage V_R. Typical values for a silicon planar epitaxial diode are $V_F = 0.8$ V at $I_F = 10$ mA (corresponding to $R_F = 80\ \Omega$) and $I_R = 0.1\ \mu$A at $V_R = 50$ V (corresponding to $R_R = 500$ M).

For small-signal operation the *dynamic*, or *incremental, resistance* r is an important parameter, and is defined as the reciprocal of the slope of the volt-ampere characteristic, $r \equiv dV/dI$. The dynamic resistance is not a constant, but depends upon the operating voltage. For example, for a semiconductor diode, we find from Eq. (6-33) that the dynamic conductance $g \equiv 1/r$ is

$$g \equiv \frac{dI}{dV} = \frac{I_o \epsilon^{V/\eta V_T}}{\eta V_T} = \frac{I + I_o}{\eta V_T} \tag{6-40}$$

For a reverse bias greater than a few tenths of a volt (so that $|V/\eta V_T| \gg 1$), g is extremely small and r is very large. On the other hand, for a forward bias greater than a few tenths of a volt, $I \gg I_o$, and r is given approximately by

$$r \approx \frac{\eta V_T}{I} \tag{6-41}$$

The dynamic resistance varies inversely with current; at room temperature and for $\eta = 1$, $r = 26/I$, where I is in milliamperes and r in ohms. For a forward current of 26 mA, the dynamic resistance is 1 Ω. The ohmic body resistance of the semiconductor may be of the same order of magnitude or even much higher than this value. Although r varies with current, in a small-signal model, it is reasonable to use the parameter r as a constant.

A Piecewise Linear Diode Characteristic A large-signal approximation which often leads to a sufficiently accurate engineering solution is the *piecewise linear* representation. For example, the piecewise linear approximation for a semiconductor diode characteristic is indicated in Fig. 6-11. The break point is not at the origin, and hence V_γ is also called the *offset*, or *threshold, voltage.* The diode behaves like an open circuit if $V < V_\gamma$, and has a constant incremental resistance $r = dV/dI$ if $V > V_\gamma$. Note that the resistance r (also designated as R_f and called the *forward resistance*) takes on added physical

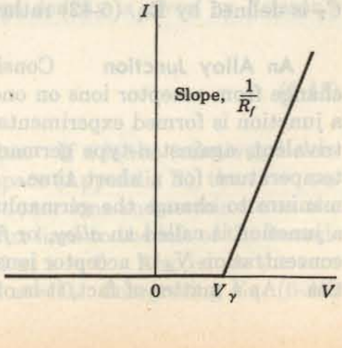


Fig. 6-11 The piecewise linear characterization of a semiconductor diode.

significance even for this large-signal model, whereas the static resistance $R_F = V/I$ is not constant and is not useful.

The numerical values V_γ and R_f to be used depend upon the type of diode and the contemplated voltage and current swings. For example, from Fig. 6-8 we find that, for a current swing from cutoff to 10 mA with a germanium diode, reasonable values are $V_\gamma = 0.6$ V and $R_f = 15\ \Omega$. On the other hand, a better approximation for current swings up to 50 mA leads to the following values: germanium, $V_\gamma = 0.3$ V, $R_f = 6\ \Omega$; silicon, $V_\gamma = 0.65$ V, $R_f = 5.5\ \Omega$. For an avalanche diode, discussed in Sec. 6-12, $V_\gamma = V_Z$, and R_f is the dynamic resistance in the breakdown region.

6-9 SPACE–CHARGE, OR TRANSITION, CAPACITANCE[1] C_T

As mentioned in Sec. 6-1, a reverse bias causes majority carriers to move away from the junction, thereby uncovering more immobile charges. Hence the thickness of the space-charge layer at the junction increases with reverse voltage. This increase in uncovered charge with applied voltage may be considered a capacitive effect. We may define an incremental capacitance C_T by

$$C_T = \left|\frac{dQ}{dV}\right| \tag{6-42}$$

where dQ is the increase in charge caused by a change dV in voltage. It follows from this definition that a change in voltage dV in a time dt will result in a current $i = dQ/dt$, given by

$$i = C_T\frac{dV}{dt} \tag{6-43}$$

Therefore a knowledge of C_T is important in considering a diode (or a transistor) as a circuit element. The quantity C_T is referred to as the *transition-region, space-charge, barrier,* or *depletion-region, capacitance.* We now consider C_T quantitatively. As it turns out, this capacitance is not a constant, but depends upon the magnitude of the reverse voltage. It is for this reason that C_T is defined by Eq. (6-42) rather than as the ratio Q/V.

An Alloy Junction Consider a junction in which there is an abrupt change from acceptor ions on one side to donor ions on the other side. Such a junction is formed experimentally, for example, by placing indium, which is trivalent, against n-type germanium and heating the combination to a high temperature for a short time. Some of the indium dissolves into the germanium to change the germanium from n to p type at the junction. Such a junction is called an *alloy,* or *fusion, junction.* It is not necessary that the concentration N_A of acceptor ions equal the concentration N_D of donor impurities. As a matter of fact, it is often advantageous to have an unsymmetrical junction. Figure 6-12 shows the charge density as a function of distance from an alloy junction in which the acceptor impurity density is assumed to be much smaller than the donor concentration. Since the net charge must be zero, then

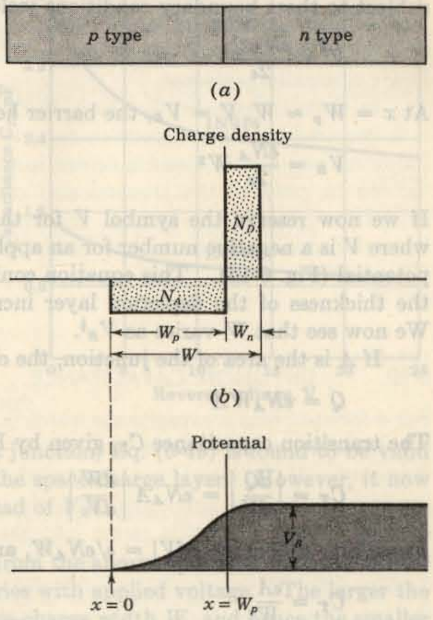


Fig. 6-12 The charge-density and potential variation at a fusion p-n junction ($W \approx 10^{-4}$ cm).

$$eN_AW_p = eN_DW_n \tag{6-44}$$

If $N_A \ll N_D$, then $W_p \gg W_n$. For simplicity, we neglect W_n and assume that the entire barrier potential V_B appears across the uncovered acceptor ions. The relationship between potential and charge density is given by Poisson's equation,

$$\frac{d^2V}{dx^2} = \frac{eN_A}{\epsilon} \tag{6-45}$$

where ϵ is the permittivity of the semiconductor. If ϵ_r is the (relative) dielectric constant and ϵ_o is the permittivity of free space (Appendix B), then $\epsilon = \epsilon_r\epsilon_o$. The electric lines of flux start on the positive donor ions and terminate on the negative acceptor ions. Hence there are no flux lines to the left of the boundary $x = 0$ in Fig. 6-12, and $\mathcal{E} = -dV/dx = 0$ at $x = 0$. Also, since the zero of potential is arbitrary, we choose $V = 0$ at $x = 0$. Integrating Eq. (6-45) subject to these boundary conditions yields

$$V = \frac{eN_Ax^2}{2\epsilon} \tag{6-46}$$

At $x = W_p \approx W$, $V = V_B$, the barrier height. Thus

$$V_B = \frac{eN_A}{2\epsilon} W^2 \tag{6-47}$$

If we now reserve the symbol V for the *applied* bias, then $V_B = V_o - V$, where V is a negative number for an applied *reverse* bias and V_o is the contact potential (Fig. 6-1*d*). This equation confirms our qualitative conclusion that the thickness of the depletion layer increases with applied reverse voltage. We now see that W varies as $V_B^{\frac{1}{2}}$.

If A is the area of the junction, the charge in the distance W is

$$Q = eN_AWA$$

The transition capacitance C_T, given by Eq. (6-42), is

$$C_T = \left|\frac{dQ}{dV}\right| = eN_AA\left|\frac{dW}{dV}\right| \tag{6-48}$$

From Eq. (6-47), $|dW/dV| = \epsilon/eN_AW$, and hence

$$C_T = \frac{\epsilon A}{W} \tag{6-49}$$

It is interesting to note that this formula is exactly the expression which is obtained for a parallel-plate capacitor of area A (square meters) and plate separation W (meters) containing a material of permittivity ϵ. If the concentration N_D is not neglected, the above results are modified only slightly. In Eq. (6-47) W represents the total space-charge width, and $1/N_A$ is replaced by $1/N_A + 1/N_D$. Equation (6-49) remains valid.

A Grown Junction A second form of junction, called a *grown junction*, is obtained by drawing a single crystal from a melt of germanium whose type is changed during the drawing process by adding first *p*-type and then *n*-type impurities. For such a grown junction the charge density varies gradually (almost linearly), as indicated in Fig. 6-13. If an analysis similar to that given above is carried out for such a junction, Eq. (6-49) is found to be valid where W equals the total width of the space-charge layer. However, it now turns out that W varies as $V_B^{\frac{1}{3}}$ instead of $V_B^{\frac{1}{2}}$.

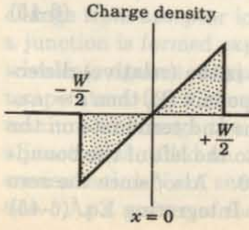


Fig. 6-13 The charge-density variation at a grown *p-n* junction.

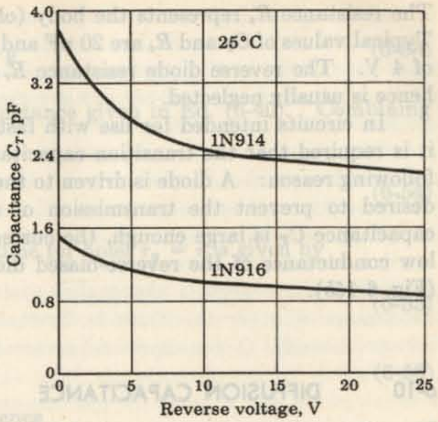


Fig. 6-14 Typical barrier-capacitance variation, with reverse voltage, of silicon diodes 1N914 and 1N916. (Courtesy of Fairchild Semiconductor Corporation.)

Varactor Diodes We observe from the above equations that the barrier capacitance is not a constant but varies with applied voltage. The larger the reverse voltage, the larger is the space-charge width W, and hence the smaller the capacitance C_T. The variation is illustrated for two typical diodes in Fig. 6-14. Similarly, for an increase in forward bias (V positive), W decreases and C_T increases.

The voltage-variable capacitance of a *p-n* junction biased in the reverse direction is useful in a number of circuits. One of these applications is voltage tuning of an *LC* resonant circuit. Other applications include self-balancing bridge circuits and special types of amplifiers, called *parametric amplifiers*.

Diodes made especially for the above applications which are based on the voltage-variable capacitance are called *varactors*, *varicaps*, or *voltacaps*. A circuit model for a varactor diode under reverse bias is shown in Fig. 6-15.

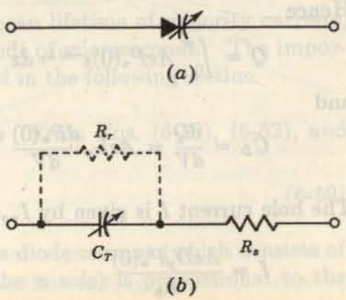


Fig. 6-15 A varactor diode under reverse bias. (*a*) Circuit symbol; (*b*) circuit model.

The resistance R_s represents the body (ohmic) series resistance of the diode. Typical values of C_T and R_s are 20 pF and 8.5 Ω, respectively, at a reverse bias of 4 V. The reverse diode resistance R_r shunting C_T is large (>1 M), and hence is usually neglected.

In circuits intended for use with fast waveforms or at high frequencies, it is required that the transition capacitance be as small as possible, for the following reason: A diode is driven to the reverse-biased condition when it is desired to prevent the transmission of a signal. However, if the barrier capacitance C_T is large enough, the current which is to be restrained by the low conductance of the reverse-biased diode will flow through the capacitor (Fig. 6-15*b*).

6-10 DIFFUSION CAPACITANCE

For a forward bias a capacitance which is much larger than that considered in the preceding section comes into play. The origin of this capacitance is now discussed. If the bias is in the forward direction, the potential barrier at the junction is lowered and holes from the p side enter the n side. Similarly, electrons from the n side move into the p side. This process of *minority-carrier injection* is discussed in Sec. 6-5, where we see that the excess hole density falls off exponentially with distance, as indicated in Fig. 6-6. The shaded area under this curve is proportional to the injected charge. As explained in Sec. 6-9, it is convenient to introduce an incremental capacitance, defined as the rate of change of injected charge with applied voltage. This capacitance C_D is called the *diffusion*, or *storage, capacitance.*

Derivation of Expressions for C_D We now make a quantitative study of the diffusion capacitance C_D. For simplicity of discussion we assume that one side of the diode, say, the p material, is so heavily doped in comparison with the n side that the current I is carried across the junction entirely by holes moving from the p to the n side, or $I = I_{pn}(0)$. The excess minority charge Q will then exist only on the n side, and is given by the shaded area of Fig. 6-6 multiplied by the diode cross section A and the electronic charge e. Hence

$$Q = \int_0^\infty AeP_n(0)\epsilon^{-x/L_p}\,dx = AeL_pP_n(0) \tag{6-50}$$

and

$$C_D = \frac{dQ}{dV} = AeL_p\frac{dP_n(0)}{dV} \tag{6-51}$$

The hole current I is given by $I_{pn}(x)$ in Eq. (6-19), with $x = 0$, or

$$I = \frac{AeD_pP_n(0)}{L_p} \tag{6-52}$$

and

$$\frac{dP_n(0)}{dV} = \frac{L_p}{AeD_p}\frac{dI}{dV} = \frac{L_p}{AeD_p}g \tag{6-53}$$

where $g \equiv dI/dV$ is the diode conductance given in Eq. (6-40). Combining Eqs. (6-51) and (6-53) yields

$$C_D = \frac{L_p{}^2g}{D_p} \tag{6-54}$$

Since from Eq. (5-51) the mean lifetime for holes $\tau_p = \tau$ is given by

$$\tau = \frac{L_p{}^2}{D_p} \tag{6-55}$$

then

$$C_D = \tau g \tag{6-56}$$

From Eq. (6-41), $g = I/\eta V_T$, and hence

$$C_D = \frac{\tau I}{\eta V_T} \tag{6-57}$$

We see that the *diffusion capacitance is proportional to the current I.* In the derivation above we have assumed that the diode current I is due to holes only. If this assumption is not satisfied, Eq. (6-56) gives the diffusion capacitance C_{D_p} due to holes only, and a similar expression can be obtained for the diffusion capacitance C_{D_n} due to electrons. The total diffusion capacitance can then be obtained as the sum of C_{D_p} and C_{D_n} (Prob. 6-30).

For a reverse bias g is very small and C_D may be neglected compared with C_T. For a forward current, on the other hand, C_D is usually much larger than C_T. For example, for germanium ($\eta = 1$) at $I = 26$ mA, $g = 1$ mho, and $C_D = \tau$. If, say, $\tau = 20$ μsec, then $C_D = 20$ μF, a value which is about a million times larger than the transition capacitance.

Despite the large value of C_D, the time constant rC_D (which is of importance in circuit applications) may not be excessive because the dynamic forward resistance $r = 1/g$ is small. From Eq. (6-56),

$$rC_D = \tau \tag{6-58}$$

Hence the diode time constant equals the mean lifetime of minority carriers, which lies in range of nanoseconds to hundreds of microseconds. The importance of τ in circuit applications is considered in the following section.

Charge-control Description of a Diode From Eqs. (6-50), (6-52), and (6-55),

$$I = Q\frac{D_p}{L_p{}^2} = \frac{Q}{\tau} \tag{6-59}$$

This very important equation states that the diode current (which consists of holes crossing the junction from the p to the n side) is proportional to the stored charge Q of excess minority carriers. The factor of proportionality is the reciprocal of the decay time constant (the mean lifetime τ) of the minority carriers. Thus, in the steady state, *the current I supplies minority carriers at the rate at which these carriers are disappearing because of the process of recombination.*

The charge-control characterization of a diode describes the device in terms of the current I and the stored charge Q, whereas the equivalent-circuit characterization uses the current I and the junction voltage V. One immediately apparent advantage of this charge-control description is that the exponential relationship between I and V is replaced by the linear dependence I on Q. The charge Q also makes a simple parameter, the sign of which determines whether the diode is forward- or reverse-biased. The diode is forward-biased if Q is positive and reverse-biased if Q is negative.

6-11 *p-n* DIODE SWITCHING TIMES

When a diode is driven from the reversed condition to the forward state or in the opposite direction, the diode response is accompanied by a transient, and an interval of time elapses before the diode recovers to its steady state. The forward recovery time t_{fr} is the time difference between the 10 percent point of the diode voltage and the time when this voltage reaches and remains within 10 percent of its final value. It turns out[5] that t_{fr} does not usually constitute a serious practical problem, and hence we here consider only the more important situation of reverse recovery.

Diode Reverse Recovery Time When an external voltage forward-biases a *p-n* junction, the steady-state density of minority carriers is as shown in Fig. 6-16*a* (compare with Fig. 6-6). The number of minority carriers is very large. These minority carriers have, in each case, been supplied from the other side of the junction, where, being majority carriers, they are in plentiful supply.

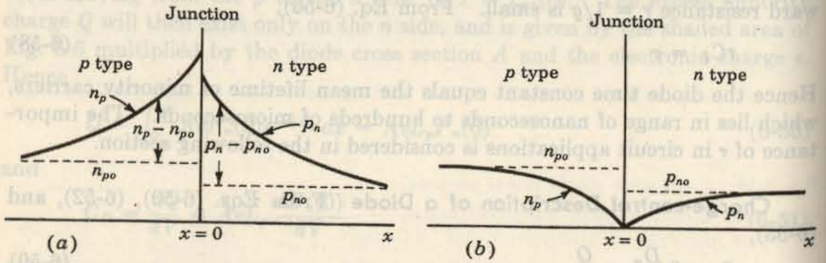


Fig. 6-16 Minority-carrier density distribution as a function of the distance x from a junction. (*a*) A forward-biased junction; (*b*) a reverse-biased junction. The injected, or excess, hole (electron) density is $p_n - p_{no}$ ($n_p - n_{po}$).

When an external voltage reverse-biases the junction, the steady-state density of minority carriers is as shown in Fig. 6-16*b*. Far from the junction the minority carriers are equal to their thermal-equilibrium values p_{no} and n_{po}, as is also the situation in Fig. 6-16*a*. As the minority carriers approach the junction they are rapidly swept across, and the density of minority carriers diminishes to zero at this junction. The current which flows, the reverse saturation current I_o, is small because the density of thermally generated minority carriers is very small.

If the external voltage is suddenly reversed in a diode circuit which has been carrying current in the forward direction, the diode current will not immediately fall to its steady-state reverse-voltage value. For the current cannot attain its steady-state value until the minority-carrier distribution, which at the moment of voltage reversal had the form in Fig. 6-16*a*, reduces to the distribution in Fig. 6-16*b*. Until such time as the *injected*, or *excess*, *minority-carrier density* $p_n - p_{no}$ (or $n_p - n_{po}$) has dropped nominally to zero, the diode will continue to conduct easily, and the current will be determined by the external resistance in the diode circuit.

Storage and Transition Times The sequence of events which accompanies the reverse biasing of a conducting diode is indicated in Fig. 6-17. We consider that the voltage in Fig. 6-17*b* is applied to the diode-resistor circuit in Fig. 6-17*a*. For a long time, and up to the time t_1, the voltage $v_i = V_F$ has been in the direction to forward-bias the diode. The resistance R_L is assumed large enough so that the drop across R_L is large in comparison with the drop across the diode. Then the current is $i \approx V_F/R_L \equiv I_F$. At the time $t = t_1$ the input voltage reverses abruptly to the value $v = -V_R$. For the reasons described above, the current does not drop to zero, but instead reverses and remains at the value $i \approx -V_R/R_L \equiv -I_R$ until the time $t = t_2$. At $t = t_2$, as is seen in Fig. 6-17*c*, the injected minority-carrier density at $x = 0$ has reached its equilibrium state. If the diode ohmic resistance is R_d, then at the time t_1 the diode voltage falls slightly [by $(I_F + I_R)R_d$] but does not reverse. At $t = t_2$, when the excess minority carriers in the immediate neighborhood of the junction have been swept back across the junction, the diode voltage begins to reverse and the magnitude of the diode current begins to decrease. The interval t_1 to t_2, for the stored-minority charge to become zero, is called the *storage time* t_s.

The time which elapses between t_2 and the time when the diode has nominally recovered is called the *transition time* t_t. This recovery interval will be completed when the minority carriers which are at some distance from the junction have diffused to the junction and crossed it and when, in addition, the junction transition capacitance across the reverse-biased junction has charged through R_L to the voltage $-V_R$.

Manufacturers normally specify the reverse recovery time of a diode t_{rr}

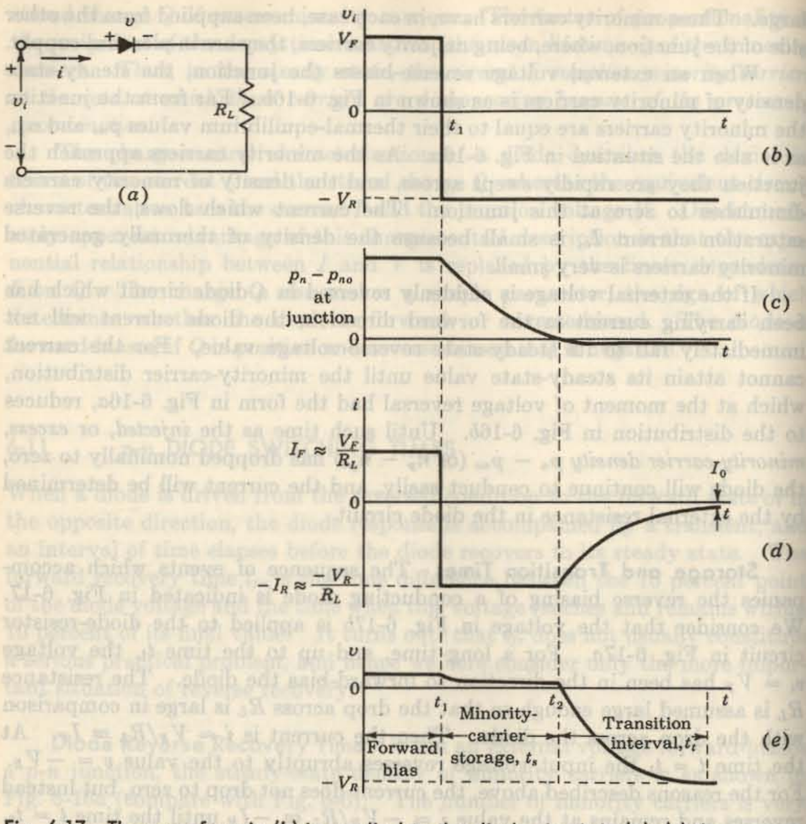


Fig. 6-17 The waveform in (b) is applied to the diode circuit in (a); (c) the excess carrier density at the junction; (d) the diode current; (e) the diode voltage.

in a typical operating condition in terms of the current waveform of Fig. 6-17*d*. The time t_{rr} is the interval from the current reversal at $t = t_1$ until the diode has recovered to a specified extent in terms either of the diode current or of the diode resistance. If the specified value of R_L is larger than several hundred ohms, ordinarily the manufacturers will specify the capacitance C_L shunting R_L in the measuring circuit which is used to determine t_{rr}. Thus we find, for the Fairchild 1N3071, that with $I_F = 30$ mA and $I_R = 30$ mA, the time required for the reverse current to fall to 1.0 mA is 50 nsec. Again we find, for the same diode, that with $I_F = 30$ mA, $-V_R = -35$ V, $R_L = 2$ K, and $C_L = 10$ pF $(-I_R = -35/2 = -17.5$ mA), the time required for the diode to recover to the extent that its resistance becomes 400 K is $t_{rr} = 400$ nsec.

Commercial switching-type diodes are available with times t_{rr} in the range from less than a nanosecond up to as high as 1 μsec in diodes intended for switching large currents.

6-12 BREAKDOWN DIODES[6]

The reverse-voltage characteristic of a semiconductor diode, including the breakdown region, is redrawn in Fig. 6-18*a*. Diodes which are designed with adequate power dissipation capabilities to operate in the breakdown region may be employed as voltage-reference or constant-voltage devices. Such diodes are known as *avalanche, breakdown*, or *Zener diodes*. They are used characteristically in the manner indicated in Fig. 6-18*b*. The source V and resistor R are selected so that, initially, the diode is operating in the breakdown region. Here the diode voltage, which is also the voltage across the load R_L, is V_Z, as in Fig. 6-18*a*, and the diode current is I_Z. The diode will now regulate the load voltage against variations in load current and against variations in supply voltage V because, in the breakdown region, large changes in diode current produce only small changes in diode voltage. Moreover, as load current or supply voltage changes, the diode current will accommodate itself to these changes to maintain a nearly constant load voltage. The diode will continue to regulate until the circuit operation requires the diode current to fall to I_{ZK}, in the neighborhood of the knee of the diode volt-ampere curve. The upper limit on diode current is determined by the power-dissipation rating of the diode.

Two mechanisms of diode breakdown for increasing reverse voltage are recognized. In one mechanism, the thermally generated electrons and holes acquire sufficient energy from the applied potential to produce new carriers by removing valence electrons from their bonds. These new carriers, in turn, produce additional carriers again through the process of disrupting bonds.

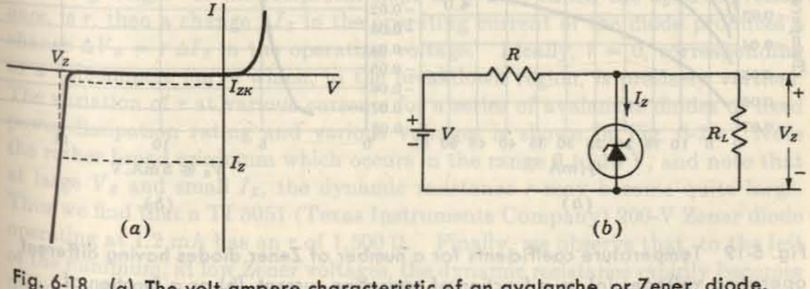
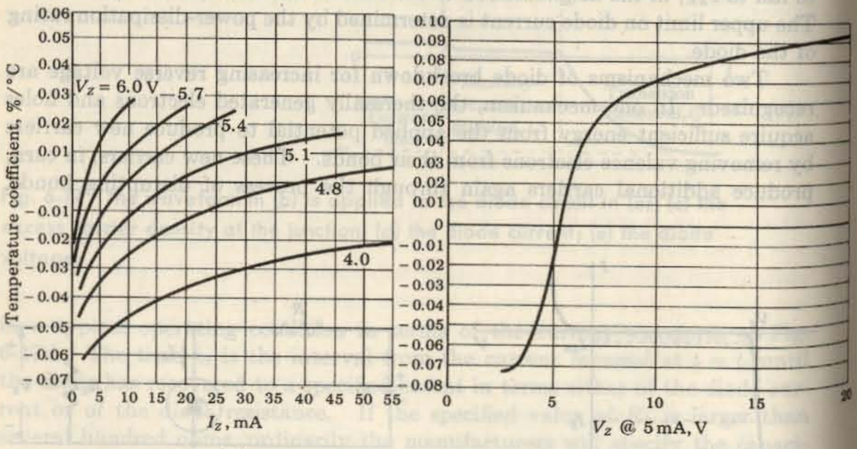


Fig. 6-18 (a) The volt-ampere characteristic of an avalanche, or Zener, diode. (b) A circuit in which such a diode is used to regulate the voltage across R_L against changes due to variations in load current and supply voltage.

This cumulative process is referred to as *avalanche multiplication*. It results in the flow of large reverse currents, and the diode finds itself in the region of *avalanche breakdown*. Even if the initially available carriers do not acquire sufficient energy to disrupt bonds, it is possible to initiate breakdown through a direct rupture of the bonds because of the existence of the strong electric field. Under these circumstances the breakdown is referred to as *Zener breakdown*. This Zener effect is now known to play an important role only in diodes with breakdown voltages below about 6 V. Nevertheless, the term *Zener* is commonly used for the *avalanche*, or *breakdown, diode* even at higher voltages. Silicon diodes operated in avalanche breakdown are available with maintaining voltages from several volts to several hundred volts and with power ratings up to 50 W.

Temperature Characteristics A matter of interest in connection with Zener diodes, as with semiconductor devices generally, is their temperature sensitivity. The temperature dependence of the reference voltage, which is indicated in Fig. 6-19*a* and *b*, is typical of what may be expected generally. In Fig. 6-19*a* the temperature coefficient of the reference voltage is plotted as a function of the operating current through the diode for various different diodes whose reference voltage at 5 mA is specified. The temperature coefficient is given as percentage change in reference voltage per centigrade degree change in diode temperature. In Fig. 6-19*b* has been plotted the temperature coefficient at a fixed diode current of 5 mA as a function of Zener voltage. The data which are used to plot this curve are taken from a series of different diodes of different Zener voltages but of fixed dissipation rating. From the curves in Fig. 6-19*a* and *b* we note that the temperature coefficients may be positive or negative and will normally be in the range ±0.1 percent/°C. Note that, if the reference voltage is above 6 V, where the physical mechanism involved is avalanche multiplication, the temperature coefficient is positive. However, below 6 V, where true Zener breakdown is involved, the temperature coefficient is negative.



(*a*) (*b*)

Fig. 6-19 Temperature coefficients for a number of Zener diodes having different operating voltages (*a*) as a function of operating current, (*b*) as a function of operating voltage. The voltage V_Z is measured at I_Z = 5 mA (from 25 to 100°C). (Courtesy of Pacific Semiconductors, Inc.)

A qualitative explanation of the sign (positive or negative) of the temperature coefficient of V_Z is now given. A junction having a narrow depletion-layer width and hence high field intensity (~10^6 V/cm even at low voltages) will break down by the Zener mechanism. An increase in temperature increases the energies of the valence electrons, and hence makes it easier for these electrons to escape from the covalent bonds. Less applied voltage is therefore required to pull these electrons from their positions in the crystal lattice and convert them into conduction electrons. Thus the Zener breakdown voltage decreases with temperature.

A junction with a broad depletion layer and therefore a low field intensity will break down by the avalanche mechanism. In this case we rely on intrinsic carriers to collide with valence electrons and create avalanche multiplication. As the temperature increases, the vibrational displacement of atoms in the crystal grows. This vibration increases the probability of collisions with the lattice atoms of the intrinsic particles as they cross the depletion width. The intrinsic holes and electrons thus have less of an opportunity to gain sufficient energy between collisions to start the avalanche process. Therefore the value of the avalanche voltage must increase with increased temperature.

Dynamic Resistance and Capacitance A matter of importance in connection with Zener diodes is the slope of the diode volt-ampere curve in the operating range. If the reciprocal slope $\Delta V_Z/\Delta I_Z$, called the *dynamic resistance*, is r, then a change ΔI_Z in the operating current of the diode produces a change $\Delta V_Z = r\,\Delta I_Z$ in the operating voltage. Ideally, $r = 0$, corresponding to a volt-ampere curve which, in the breakdown region, is precisely vertical. The variation of r at various currents for a series of avalanche diodes of fixed power-dissipation rating and various voltages is shown in Fig. 6-20. Note the rather broad minimum which occurs in the range 6 to 10 V, and note that at large V_Z and small I_Z, the dynamic resistance r may become quite large. Thus we find that a TI 3051 (Texas Instruments Company) 200-V Zener diode operating at 1.2 mA has an r of 1,500 Ω. Finally, we observe that, to the left of the minimum, at low Zener voltages, the dynamic resistance rapidly becomes quite large. Some manufacturers specify the minimum current I_{ZK} (Fig. 6-18*a*) below which the diode should not be used. Since this current is on the knee of the curve, where the dynamic resistance is large, then for currents

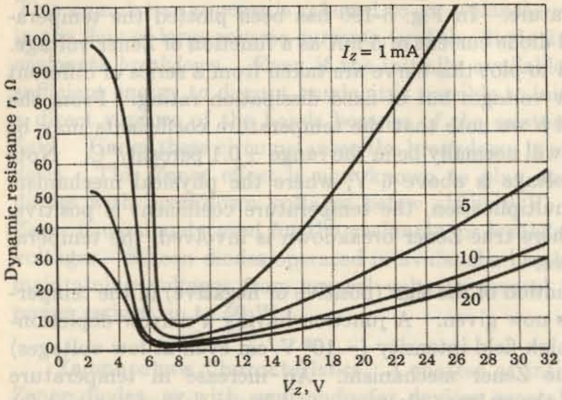


Fig. 6-20 The dynamic resistance at a number of currents for Zener diodes of different operating voltages at 25°C. The measurements are made with a 60-Hz current at 10 percent of the dc current. (Courtesy of Pacific Semiconductors, Inc.)

lower than I_{ZK} the regulation will be poor. Some diodes exhibit a very sharp knee even down into the microampere region.

The capacitance across a breakdown diode is the transition capacitance, and hence varies inversely as some power of the voltage. Since C_T is proportional to the cross-sectional area of the diode, high-power avalanche diodes have very large capacitances. Values of C_T from 10 to 10,000 pF are common.

Additional Reference Diodes Zener diodes are available with voltages as low as about 2 V. Below this voltage it is customary, for reference and regulating purposes, to use diodes in the *forward* direction. As appears in Fig. 6-8, the volt-ampere characteristic of a forward-biased diode (sometimes called a *stabistor*) is not unlike the reverse characteristic, with the exception that, in the forward direction, the knee of the characteristic occurs at lower voltage. A number of forward-biased diodes may be operated in series to reach higher voltages. Such series combinations, packaged as single units, are available with voltages up to about 5 V, and may be preferred to reverse-biased Zener diodes, which at low voltages, as seen in Fig. 6-20, have very large values of dynamic resistance.

When it is important that a Zener diode operate with a low temperature coefficient, it may be feasible to operate an appropriate diode at a current where the temperature coefficient is at or near zero. Quite frequently, such operation is not convenient, particularly at higher voltages and when the diode must operate over a range of currents. Under these circumstances temperature-compensated avalanche diodes find application. Such diodes consist of a reverse-biased Zener diode with a positive temperature coefficient, combined in a single package with a forward-biased diode whose temperature coefficient is negative. As an example, the Transitron SV3176 silicon 8-V reference diode has a temperature coefficient of ±0.001 percent/°C at 10 mA over the range −55 to +100°C. The dynamic resistance is only 15 Ω. The temperature coefficient remains below 0.002 percent/°C for currents in the range 8 to 12 mA. The voltage stability with time of some of these reference diodes is comparable with that of conventional standard cells.

When a high-voltage reference is required, it is usually advantageous (except of course with respect to economy) to use two or more diodes in series rather than a single diode. This combination will allow higher voltage, higher dissipation, lower temperature coefficient, and lower dynamic resistance.

6-13 THE TUNNEL DIODE

A *p-n* junction diode of the type discussed in Sec. 6-1 has an impurity concentration of about 1 part in 10^8. With this amount of doping, the width of the depletion layer, which constitutes a potential barrier at the junction, is of the order of 5 microns (5×10^{-4} cm). This potential barrier restrains the flow of carriers from the side of the junction where they constitute majority carriers to the side where they constitute minority carriers. If the concentration of impurity atoms is greatly increased, say, to 1 part in 10^3 (corresponding to a

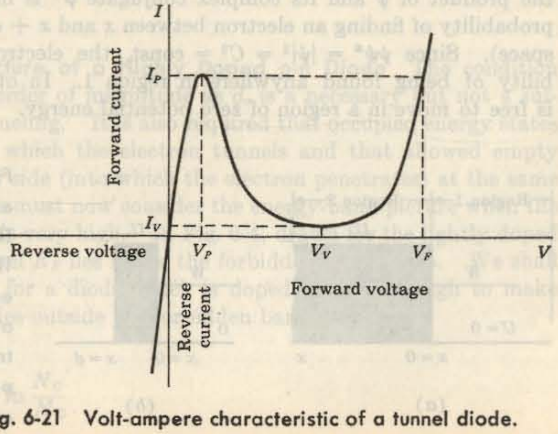


Fig. 6-21 Volt-ampere characteristic of a tunnel diode.