Therefore, $4 \times N_C e^{-8} = N_C e^{-40(E_C - E_{F1})}$

Therefore, $4 = e^{-40(E_C - E_{F1})+8}$

Taking natural logarithm on both sides, we get

$$\ln 4 = -40\,(E_C - E_{F1}) + 8$$

$$1.386 = -40(E_C - E_{F1}) + 8$$

Therefore, $E_C - E_{F1} = 0.165$ eV

(b) When $N_D = 8N_{DO}$ and $E_F = E_{F2}$, then

$$\ln 8 = -40\,(E_C - E_{F2}) + 8$$

$$2.08 = -40\,(E_C - E_{F2}) + 8$$

Therefore, $E_C - E_{F2} = 0.148$ eV

Example 4.5 In a P-type semiconductor, the Fermi level lies 0.4 eV above the valence band. Determine the new position of Fermi level if the concentration of acceptor atoms is multiplied by a factor of (a) 0.5 and (b) 4. Assume $kT = 0.025$ eV.

Solution: In a P-type material, the concentration of acceptor atoms is given by

$$N_A = N_V\, e^{-(E_F - E_V)/KT}$$

Let initially $N_A = N_{AO}$, $E_F = E_{FO}$ and $E_{FO} - E_V = 0.4$eV

Therefore, $N_{AO} = N_V\, e^{-0.4/0.025} = N_V e^{-16}$

(a) When $N_A = 0.5,\ N_{AO}$ and $E_F = E_{F1}$, then

$$0.5N_{AO} = N_V\, e^{-(E_{F1} - E_V)/0.025} = N_V e^{-40(E_{F1} - E_V)}$$

Therefore, $0.5 \times N_V\, e^{-16} = N_V e^{-40(E_{F1} - E_V)}$

Therefore, $0.5 = e^{-40(E_{F1} - E_V)+16}$

Taking natural logarithm on both sides, we get

$$\ln(0.5) = -40(E_{F1} - E_V) + 16$$

Therefore, $E_{F1} - E_V = 0.417$ eV

(b) When $N_A = 4N_{AO}$ and $E_F = E_{F2}$, then

$$\ln 4 = -40(E_{F2} - E_V) + 16$$

Therefore, $E_{F2} - E_V = 0.365$ eV

4.5 MASS-ACTION LAW

If a pure semiconductor is doped with N-type impurities, the number of electrons in the conduction band increases above a level and the number of holels in the valence band decreases below a level, which would be available in the intrinsic (pure) semiconductor. Similarly, the addition of P-type impurities to a pure semiconductor increases the number of holes in the valence band above a level and decreases the number of electrons in the conduction band below a level, which would have been available in the intrinsic semiconductor. This is because the rate of recombination increases due to the presence of a large number of free electrons (or holes).

Further, the experimental results state that under thermal equilibrium for any semiconductor, the product of the number of holes and the number of electrons is constant and is independent of the amount of donor and acceptor impurity doping. This relation is known as *mass-action law* and is given by

$$n.p = n_i^2 \tag{4.3}$$

where n is the number of free electrons per unit volume, p the number of holes per unit volume and n_i the intrinsic concentration.

While considering the conductivity of the doped semiconductors, only the dominant majority charge carriers have to be considered.

Charge densities in N-type and P-type semiconductors The law of mass-action has given the relationship between free electron concentration and hole concentration. These concentrations are further related by the law of Electrical Neutrality as explained below.

Let N_D be the concentration of donor atoms in an N-type semiconductor. In order to maintain the electric neutrality of the crystal, we have

$$n_N = N_D + p_N$$
$$\approx N_D$$

where n_N and p_N are the electron and hole concentration in the N-type semiconductor. The value of p_N is obtained from the relations of mass-action law as

$$p_N = \frac{n_i^2}{n_N}$$

$$\approx \frac{n_i^2}{N_D}, \text{ which is} \ll n_N \quad \text{or} \quad N_D.$$

Similarly, in a P-type semiconductor we have

$$p_P = N_A + n_P$$
$$\approx N_A$$

From mass-action law, $n_P = \dfrac{n_i^2}{p_P}$

Therefore, $n_P = \dfrac{n_i^2}{N_A}$, which is $\ll p_P$ or N_A

where N_A, p_P and n_P are the concentrations of acceptor impurities, holes and electrons respectively in a P-type semiconductor.

Extrinsic conductivity The conductivity of an N-type semiconductor is given by

$$\sigma_N = qn_N \mu_n \approx qN_D \mu_n, \text{ since } n_N \approx N_D.$$

The conductivity of a P-type semiconductor is given by

$$\sigma_P = qp_P \mu_P \approx q N_A \mu_P, \text{ since } p_P \approx N_A.$$

The doping of intrinsic semiconductor considerably increases its conductivity.

If the concentration of donor atoms added to a P-type semiconductor exceeds the concentration of acceptor atoms, i.e. $N_D \gg N_A$, then the semiconductor is converted from a P-type to N-type. Similarly, a large number of acceptor atoms added to an N-type semiconductor can convert it to a P-type semiconductor if $N_A >> N_D$. This concept is precisely used in the fabrication of PN junction, which is an essential part of semiconductor devices and integrated circuits.

Example 4.6 Find the conductivity of silicon (a) in intrinsic condition at a room temperature of 300 °K, (b) with donor impurity of 1 in 10^8, (c) with acceptor impurity of 1 in 5×10^7 and (d) with both the above impurities present simultaneously. Given that n_i for silicon at 300 °K is $1.5 \times 10^{10}\ \text{cm}^{-3}$, $\mu_n = 1300\ \text{cm}^2/\text{V-s}$, $\mu_p = 500\ \text{cm}^2/\text{V-s}$, number of Si atoms per $\text{cm}^3 = 5 \times 10^{22}$.

(a) In intrinsic condition, $n = p = n_i$

$$\text{Hence, } \sigma_i = qn_i(\mu_n + \mu_p)$$
$$= (1.6 \times 10^{-19})(1.5 \times 10^{10})(1300 + 500)$$
$$= 4.32 \times 10^{-6}\ \text{S/cm}$$

(b) Number of silicon atoms/$\text{cm}^3 = 5 \times 10^{22}$

$$\text{Hence, } N_D = \frac{5 \times 10^{22}}{10^8} = 5 \times 10^{14}\ \text{cm}^{-3}$$

Further, $n \approx N_D$

$$\text{Therefore, } \quad p = \frac{n_i^2}{n} \approx \frac{n_i^2}{N_D}$$
$$= \frac{(1.5 \times 10^{10})^2}{5 \times 10^{14}} = 0.46 \times 10^6\ \text{cm}^{-3}$$

Thus $p << n$. Hence p may be neglected while calculating the conductivity.

$$\text{Hence, } \quad \sigma = nq\mu_n = N_D q \mu_n$$
$$= (5 \times 10^{14})(1.6 \times 10^{-19})(1300)$$
$$= 0.104\ \text{S/cm.}$$

$$\text{(c) } N_A = \frac{5 \times 10^{22}}{5 \times 10^7} = 10^{15}\ \text{cm}^{-3}$$

Further, $p \approx N_A$

$$\text{Hence, } \quad n = \frac{n_i^2}{p} \approx \frac{n_i^2}{N_A}$$
$$= \frac{(1.5 \times 10^{10})^2}{10^{15}} = 2.25 \times 10^5\ \text{cm}^{-3}$$

Thus, $p >> n$. Hence n may be neglected while calculating the conductivity.

$$\text{Hence, } \quad \sigma = pq\mu_P = N_A q \mu_P$$
$$= (10^{15} \times 1.6 \times 10^{-19} \times 500)$$
$$= 0.08\ \text{S/cm.}$$